ENHANCING SERVICE CAPABILITY OF HETEROGENEOUS QUEUEING MODEL WITH VACATION POLICY

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ABSTRACT: This investigation is concerned with the study of bulk service queueing model of two heterogeneous servers with different service rates for two servers (fast and slow). It is assumed to be mutually liberated and exponentially distributed with parameters μ_1 and μ_2 respectively. The arrivals are served in batches in Poisson fashion with parameter λ according to FCFS discipline. Vacation strategy for slow server is deliberated in this model. The steady state results and the structure features are derived for this model.

KEY WORDS: Bulk queue, Heterogeneous servers, Single Vacation.

I. INTRODUCTION

Many real world complex problems can be solved using the some of the effective ways from nature. The queuing theory is a beneficial statistical technique for solving that type of peculiar problems. The queueing models have professed an excellent growth in the applications of bulk queues to many stuffing situations. The primary reason for analyzing is that bulk queueing models are often faced in real life systems such as computer system, telecommunication as well as industrial evolutions such as production/ inventory systems etc. The queueing theory is also known as the theory of congestion. It is a branch of operational research that determines the relationship between demand on a service system and the delays lamented by the users of that system. It is measured by one of the standard methodologies with simulation, linear programming and management learning.

There is a significant attention was paid to analyze the queueing system with various vacation policies. Many investigators have analyzed vacation queueing problems for both Markovian and Non-Markovian queueing models. The Markovian models may be more appropriate in practice and have influenced advantages. The study of bulk queues was invented by Harris C M & Gross D (1974). The detailed complete survey on queueing systems with server vacations and priority systems are investigated by Doshi. B T (1986), Borthakur. A & Choudhury.

G (1997), Bruneel. H and Wuyts. I (1994). Its wide range of practical applications in a variety of situations makes it attractive to many researchers to work on various queueing models and to obtain closed form solutions.

A queuing theory model based on Poisson Probability Distribution can be designed to determine the parameters such as congestion, waiting period, delay. Some factors such as appropriate number of judges, establishment of new courts etc. that cause delay in the process should be solved so that the efficiency of the model can be increased. This type of queueing models are classified by modeling the techniques of cloud computing using the queuing theory in seven categories based on their focus area: (1) performance, (2) quality of service, (3) workflow scheduling, (4) energy savings, (5) resource management, (6) priority-based servicing, and (7) reliability.

The rule with a fixed batch size *n* has been considered by Medhi. J and Borthakur. A (1972) and others. Baily (1954) introduced the queueing process with bulk service with a rule that if immediately after the completion of a service, if the server finds less than '*a*' units present, the server waits until there are '*a*' units, where upon the server takes the batch of '*a*' units for service; if the server finds '*a*' or more but atmost '*b*', it takes them all in the batch and if the server finds more than '*b*', it takes in the batch for service '*b*' units, while others wait. The batch takes a minimum of '*a*' units and a maximum of '*b*' units. This rule will be called general bulk service rule.

Choudhury. G and Madan. KC (2006), applied the same regulation with the restriction that $n \neq 0$ ($1 \leq n \leq b$), that is the service facility stops until '*a*' unit arrives. Which is called as an usual bulk service rule, while Bailey; rule will be its modified type named as bulk service rule with occasionally available server and found that the distribution of the queue length for the improved rule can be obtained from that of the usual rule.

Borthakur. A, Medhi. J (1972) worked on a two server with a common bulk service rule. A $M/M(a,b,c)/2$ bulk queueing model for non-identical servers with vacation has been discussed by Mishra and Pand ey (2002) and derived the stationary circulations of the digit of customers and other performance measures. Krishnamoorthi B (1963) has studied the heterogeneity (fast and slow) servers problem. Due to fundamental assumptions, he has shown that the fast server should be always used for service, and the slow server should be only used when the fast server is busy and the number of customers waiting in the queue exceeds a certain threshold. A bulk queueing model *M/M(a,b)/(2,1)* for non-identical servers with delayed vacation is studied by Sree Parimala. R and Palaniammal. S (2014).

II. PRESENTATIONS OF THE MODEL

The projected queueing model has distended the applications in diverse practical systems dealing with human style including healthcare and private business firms at which customers may arrive in groups. At the end of busy periods, the fast server always retained in a system. Once the vacation period is ended, the slow server shifts to the busy period if there are sufficient customers in the queue; otherwise it may take a fixed consecutive vacations, at the end of the successive vacations, the server switches to busy period and stays idle or busy depending on the availability of the customers in the system. During the vacation period, a customer may leave the system whenever its waiting time is longer than the customer's patience time.

Another practical application of the planned model arises in communication organizations: It is broadly familiar that the impatience phenomenon is one of the determining features for the performance of call centers. From a business point of view, a call center is an entity that syndicates voice and data communication technologies, permitting a company to implement dangerous business strategies in order to decrease costs and increase profits. It is typically set up for sales, marketing, technical care, and patron service purposes.

The present investigation, an attempt has been made to analyze two heterogeneous *M/M(a,b)/(2,1)* queueing system with different service rate in which the fast server is always available in the system for service and the slow server can avail the single vacation if there are less than '*a-1*' customers in the queue. That is, single vacation for slow server is discussed. The study of this queueing model is organized as follows. The model is described in Section 3. Queueing model is formulated mathematically along with notations in Section 4. The steady state solutions have been obtained in Section 5. The performance measures and mean queue length are derived in Section 6. Concluding remarks and notable features of investigation done are highlighted in Section 7.

III. PROBLEM DESCRIPTION

In this model it is assumed that the arrival pattern is Poisson with parameter λ. Service is done in batches according to the general bulk service rule introduced by Neuts (1967). The late arrivals are not allowed to join the ongoing service. The successive service times are assumed to be mutually independent and exponentially distributed with parameters μ_i (i = 1, 2) for each of the two servers (fast and slow) and $\mu_1 > \mu_2$.

On completing the service if the slow server finds less than '*a* ' or '*a* – 1' customers in the queue and the fast server is busy, he wait in the system for a random period of time before going for vacation, which is called delayed time and goes for single vacation which is exponentially distributed with a parameter θ and on returning from vacation, if the slow server finds less than '*a*' waiting customers and the fast server is busy or idle in the system, he stay in the system until he finds minimum '*a*' number of customers. i.e., in this scheme single vacation for slow server is also considered. These characteristics analysis is mainly used in cloud computing and data centers.

The model leads to the state space

$\{(i, j, n);\ i, j = 0,1,\ i+j \neq 2,\ 0 \leq n \leq a-1\}\ U\ \{(1, 1, M, n);\ n \geq 0\}\ U\ \{(1, 0, M, n);\ n \geq a, M \leq b\}$

In the state *(i, j, n)*,the index for this model are,

(i) $i=0$ denotes to the situation that the fast server is idle

(ii) $i=1$ means that the fast server is busy

(iii) $j=0$ states to the state that the slow server is on vacation

(iv) $j=1$ that the slow server is busy.

(v) $n \geq 0$ refers to the number of waiting customers in the queue

(vi) $M \leq b$ refers the components in service.

IV. STEADY STATE EQUATIONS

Defining $p_{i,j,n}(t)$ as the probability that the system is in the state (i, j, n), $i, j = 0,1$, $n \geq 0$, $M \leq b$ and assuming that the steady state probabilities exists, the balance equation in the steady state are given by

$$((M-n-1)\lambda + \theta)\, P_{100} = (M-1)\lambda P_{idle\ 0\ a-1} + \mu_2 P_{110} + \mu_1 \sum_{i=a}^{b} P_{10\ idle} \quad (1)$$

$$(M\lambda + \mu_1)\, P_{11n} = (M-n-1)\lambda P_{11n-1} + \mu_1 P_{11n+b} + \alpha\mu_2 P_{11n+b} + \theta P_{10n+b} \quad (n \geq 1) \quad (2)$$

$$((M-1)\lambda + \mu_1 + \theta)\, P_{10n} = M\lambda\, P_{10n-1} + \mu_1 P_{10n+b} \quad (n \geq a) \quad (3)$$

$$(M\lambda + \mu_1 + \alpha\,\mu_2)\, P_{110} = (\mu_1 + \mu_2) \sum_{i=a}^{b} P_{11\ idle} + \theta \sum_{i=a}^{b} P_{10\ idle} + M\lambda P_{1\ idle\ a-1} \quad (4)$$

$$((M-1)\lambda + \mu_1)\, P_{1\ idle\ n} = (M-n-1)\,\lambda\, P_{1\ idle\ n-1} + \theta\, P_{1\ 0\ n} \quad (0 \leq n \leq a-1) \quad (5)$$

$$((M-2)\lambda + \mu_1)\, P_{1\,idle\,0} = \theta\, P_{100} + (M-1)\lambda\, P_{1\,idle\,a-1} \quad (6)$$

$$(M\lambda + \alpha\mu_2)\, P_{idle\,10} = \mu_1 P_{110} \quad (7)$$

$$((M-n-1)\lambda + \alpha\mu_2)\, P_{idle\,1n} = (M-2)\lambda\, P_{idle\,1n-1} + \mu_1 P_{11n} \qquad (0 \le n \le a-1) \quad (8)$$

$$((M-n-1)\lambda + \theta)\, P_{idle\,0n} = \mu_1 P_{10n} + \alpha\mu_2 P_{idle\,1n} + M\lambda\, P_{idle\,1\,n-1} (0 \le n \le a-1) \quad (9)$$

$$M\lambda\, P_{idle\,idle\,0} = \mu_1 P_{1\,idle\,0} \quad (10)$$

V. COMPUTATION OF STEADY STATE SOLUTIONS

Let E denote the forward shifting operative, that defined by E(P_{10n}) = $P_{10n+1.}$ From equation (3) implies($\mu_1 E^{b+1}$- ((M- 1)$\lambda + \mu_1 + \theta$)E+$M\lambda$) P_{10n} = 0 $(n \ge a-1)$. The characteristic equation has only one real root inside the circle $|Z|$ =1, then we know that by Rouche's theorem when $\rho = \frac{\lambda+\theta}{b\mu_1}$ is less than 1.If r_0 (say) is the root of the above characteristic equation with $|r_0|<1$,

Then $p_{10n} = A_1 r_0^n$, (n ≥ a), is the solution for the homogeneous difference equation (3), we have

$$P_{10n} = r_0^{n-a+1} P_{10a-1} (n \ge a) \quad (11)$$

Using equation (2), we get,

$$(\mu_1 + \mu_2)\, E^{b+1} - ((M - n - 1)\lambda + \mu_1)E + M\lambda)\, P_{11n} = -\theta\, P_{10n+b+1} (M \le b, n \ge 0)$$

The characteristic equation of this equation has only one real root r_1 by Rouche's theorem which lies in the interval (0,1) , ρ< 1, where$\rho = \frac{(M-n-1)\lambda}{b(\mu_1+\mu_2)}$ and after simplification

$$P_{11n} = (A_2 r_1^n + k_1 r_0^n) P_{10a-1} (n \ge 0) \quad (12)$$

Where A_2 is a constant and$k_1 = \frac{-\theta\mu_1 r_0^{b-a+2}}{\mu_2[((M-n-1)\lambda+\theta)r_0 - M\lambda] + \theta r_0 \mu_1}$

From equation (5), $(\mu_1\, E^{b+1} - ((M-1)\lambda + \mu_1)E + M\lambda)\, P_{1idelen} = 0$ for $(0 \le n \le a-2)$

The above derived equation has only one real root r_2 , by Rouche's theorem it lies in the intermission (0,1), ρ< 1, where $\rho = \frac{(M-n-1)\lambda + \theta}{b\mu_1}$ and after simplification, we get

$P_{1idlen} = (A_3 r_2^n + k_2 r_1^n + k_3 r_0^n)\, P_{10a-1}(1 \le n \le a-1, M > 0)$ (13)

where $k_2 = \frac{(\mu_1+\mu_2)A_2 r_1}{((\lambda-\mu_1-\theta)r_1)\mu_2+\theta r_1}$ and $k_3 = -\mu_2 k_1$

Solving equation (13), $(((\mathrm{M}-1)\lambda+\mu_2)E - M\lambda)\, P_{Idle\ 1n} = \mu_1 P_{11n+1}(1 \le n \le a-1)$

$P_{Idle\ 1n} = (A_4 r_3^n + A_2 k(r_1) r_1^n + k_1 k(r_0) r_0^n)\, P_{10a-1}(1 \le n \le a-1)$ (14)

here $r_3 = \frac{(\mathrm{M}-1)\lambda}{M\lambda+\mu_1}$ and $k(x) = \frac{\mu_1 x}{(\lambda+\mu_2)x - M\lambda}$

Solving equation (8)

$P_{Idle\ 0n} = (A_5 r_5^n + A_4 G(r_3) r_3^n + A_3 G(r_2) r_2^n]\, r_1^n$

$+ [k(r_0)G(r_0) + B(r_0)] r_0^n)\, P_{10a-1}(1 \le n \le a-1)$ (15)

here $r_5 = \frac{M\lambda}{(\mathrm{M}-\mathrm{n})\lambda+\theta}$, $G(x) = \frac{x\mu_2}{(\lambda+\theta)x-M\lambda}$, $B(x) = \frac{x\mu_1}{(\theta+(\mathrm{M}-\mathrm{n}-1)\lambda)x-M\lambda}$

Substituting the standards of $P_{1\ Idle\ n}$, $P_{Idle\ 0n}$, P_{10n} and P_{11n} in equation (9),

$((\mathrm{M}-\mathrm{n}-1)\lambda+\mu_1+\mu_2)(A_2)P_{10a-1} = (\mu_1+\mu_2)\left[A_2 \frac{r_1^a - r_1^{b+1}}{1-r_1}\right] + \theta\, [A_3 \frac{r_2^a - r_2^{b+1}}{1-r_2} + k_2 \frac{r_1^a - r_1^{b+1}}{1-r_1} +$

$k_3 \frac{r_0^a - r_0^{b+1}}{1-r_0} \Big] P_{10a-1}$ (16)

After simplification, we obtain $A_3 = A_2 L(r_1) + L(r_0)$, (17)

where $L(x) = \frac{1-r_1}{1-r_1^a}\left[\frac{\theta x}{(\mu_1)(1-x)M\lambda} - \frac{B(x)((\mathrm{M}-\mathrm{n}-1)\lambda+\mu_1)(1-x^a)}{(1-x)}\right]$

The value of A_4 can be determined using the equation (14)

$A_4 = \frac{1}{r_3^{a-1}}\, [A_2(1 - k(r_1) r_1^{a-1}) - k_1 k(r_0) r_0^{a-1}]$ (18)

Also, we obtain the value of A_5

$A_5 = \frac{1}{r_4^{a-1}}[A_3(1 - B(r_2) r_2^{a-1}) + (\mathrm{M}-\mathrm{n}-1)\lambda - k_2 B(r_1) r_1^{a-1} - k_3 B(r_0) r_0^{a-1}]$ (19)

The value of A_2 is. $A_2 = \frac{B(r_2)\left(\frac{1-r_2^{a-1}}{1-r_2}\right) + k(r_1)\left(\frac{1-r_1^{a-1}}{1-r_1}\right) - \frac{\theta\rho\, r_{0(\mathrm{M}-\mathrm{n}-1)\lambda}}{(M\lambda+\theta)(1-r_0)}}{\frac{\theta\rho\, r_1(\mathrm{M}-\mathrm{n}-1)\lambda}{(M\lambda+\theta)(1-r_1)} - B(r_1)\left(\frac{1-r_1^{a-1}}{1-r_1}\right) - k(r_0)\left(\frac{1-r_0^{a-1}}{1-r_0}\right)}$, where $\rho = \frac{(\mathrm{M}-\mathrm{n}-1)\lambda\theta}{\mu_1+\mu_2}$ (20)

Exhausting the value of A_2, we can compute the values of A_3, A_4 and A_5, All the steady state probabilities are obtained in terms of P_{10a-1}.

The value of P_{10a-1} can be determined using normalizing condition,

$$\sum_{n=0}^{a-1}(P_{10n}+P_{Idle\ 1\ n}+ P_{1\ idle\ n} + P_{Idle\ 0\ n})+\sum_{n=a}^{\infty} P_{10n}+\sum_{n=0}^{\infty} P_{11n} =1 \tag{21}$$

we get the solution after some algebraic simplification as follows,

$$\sum_{n=0}^{a-1}(P_{10n}+P_{Idle\ 1\ n}+P_{1\ idle\ n} + P_{Idle\ 0\ n}) = [A_5M(r_4)+A_4[G(r_3)\, M(r_3) +K(r_3)N(r_3)]+A_3[M(r_2)G(r_2) + N(r_2)K(r_2)]+A_2[k_1G(r_1)M(r_1)+k_3\mathrm{B}(r_1)K(r_1)] +k_1K(r_0))+ k_2\ \mathrm{B}(r_0)]\ P_{10a-1} \tag{22}$$

Where, $M(x) = \left(\frac{1-x^a}{1-x}\right) + \frac{\mu_1}{(1-M\lambda)}\left(\frac{a}{1-x} - \frac{x(1-x^a)}{(1-x)^2}\right)$,

$$N(x) = \left(\frac{1-x^a}{1-x}\right) + \frac{\mu_2}{(1-(\mathrm{M}-\mathrm{n}-1)\lambda)}\left(\frac{a}{1-x} - \frac{x(1-x^a)}{(1-x)^2}\right)$$

then $$P_{10a-1}{}^{-1} = [A_6M(r_5) + A_5M(r_4) + A_4[G(r_3)\, M(r_3) + K(r_3)N(r_3)] +A_3[G(r_2)\, M(r_2)+K(r_2)N(r_2)] + A_2[k_1G(r_1)\, M(r_1)+ k_3\mathrm{B}(r_1)K(r_1)] + \frac{r_1}{1-r_1} + \frac{\mathrm{B}(r_1)}{1-r_1} +k_1K(r_0)+ k_2\ \mathrm{B}(r_0)] + \frac{r_0}{1-r_0} + \frac{\mathrm{B}(r_0)}{1-r_0} \tag{23}$$

VI. PERFORMANCE MEASURES

The effectiveness of the queueing scheme can be proved by judgement the performance measures of the queueing systems under deliberation. As the steady-state probabilities are known, various performance measures of the queue can be easily obtained.

(a) Mean Queue Length

Let L_q be the expected number of customers in the queue then

$$L_q=\sum_{n=0}^{a-1} n(P_{10n} + (\mathrm{M}-\mathrm{n}-1)P_{Idle\ 1\ n} + \mathrm{M}P_{1\ idle\ n} + P_{Idle\ 0\ n}) + \sum_{n=a}^{\infty} nP_{10n} + \sum_{n=1}^{M} nP_{11n} \tag{24}$$

(b) Probability that both the servers are busy (P_{2B})

$$P_{2B} = (A_2\frac{1}{1-r_1} + Mk_1\frac{1}{1-r_0})P_{10a-1} \tag{25}$$

(c)Probability that one server is busy (P_{1B})

$$P_{1B}=[(\mathrm{M}-\mathrm{n}-1)A_5\frac{r_4}{1-r_4}+A_3\frac{r_3}{1-r_3}+Mk_2\frac{r_1}{1-r_1}+k_3\frac{r_0}{1-r_0}]P_{10a-1} \tag{26}$$

(d) Probability that the servers either on vacation or idle (P_{idleB})

$$P_{Idle\ B} = [A_4 \frac{r_3}{1-r_3} + MA_3 \frac{1-r_2^n}{r_2} + k_1 k(r_0) + (\text{M} - \text{n} - 1)k_3 \frac{r_0}{1-r_0}]P_{10a-1} \quad (27)$$

VII. CONCLUSION

In this present study, a Markovian bulk queueing model by heterogeneous servers with single vacation for slow server depends on batch size are measured. All the performance measures are established in this model. This present study satisfies the purpose because, the fast server is always retained in the system and slow server can go only for a single vacation. The model proposed here is applied for many real world problems.

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